Mathematical Appendix to Rohde & Schwarz Application Note 1GP45

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Oversampling for ARB with Interpolation Filter

Let W_I be the bandwidth of the interpolation filter and W_S the bandwidth of the modulated signal. To avoid cutting the signal with the interpolation filter:

$$W_I \geq W_S \tag{1}$$

This equation can be written as:

$$\frac{O \cdot f_{sym}}{f_{sample}} \cdot W_I \geq W_S \tag{2}$$

with f_{sample} being the sample rate, O the oversampling factor and f_{sym} the sample rate of the signal. (Remember that $f_{sample} = O \cdot f_{sym}$) This gives:

$$O \cdot \frac{W_I}{f_{sample}} \geq \frac{W_S}{f_{sym}} \tag{3}$$

For a W-CDMA signal with a $\sqrt{\cos}$ filter, $\alpha = 0.22$:

$$\frac{W_S}{f_{sym}} = \frac{1+\alpha}{2} = 0.61 \tag{4}$$

The interpolation filter has the standardized bandwidth:

$$\frac{W_I}{f_{sample}} = 0.375 \tag{5}$$

This gives:

$$O \geq \frac{0.61}{0.375} = 1.63 \tag{6}$$

Effect of non-ideal I/Q Signals

We will discuss this for a single CW carrier with an offset from the RF center frequency, i.e. at $\omega_0 + \omega_M$.

Ideal I/Q Signal

The ideal I/Q signal for this scenario is:

$$I(t) = \cos \omega_M t \quad (7)$$
$$Q(t) = \sin \omega_M t \quad (8)$$

Then - if we assume that the I/Q modulator itself is ideal - the modulated RF signal will be:

$$\begin{aligned} s(t) &= \Re\left\{ (I(t) + \imath Q(t))\, e^{\imath\omega_0 t} \right\} \\ &= \cos \omega_M t \cdot \cos \omega_0 t - \sin \omega_M t \cdot \sin \omega_0 t \\ &= \cos\left(\omega_0 + \omega_M\right) t \end{aligned} \quad (9)$$

Non-ideal I/Q signal

In reality both the I/Q Modulator and the I/Q input signal are not ideal. This can be described as small deviations in amplitude and phase of the Q signal:

$$I(t) = \cos \omega_M t \quad (10)$$
$$Q(t) = (1 + \epsilon) \sin\left(\omega_M t + \varphi\right) \quad (11)$$

with $\epsilon \ll 1, \varphi \ll 1$. For φ the following approximations are valid:

$$\sin \varphi \approx \varphi, \quad \cos \varphi \approx 1 \quad (12)$$

ϵ can result either from different magnitudes for I and Q of the input signal, or from different gain in the I and Q channel of the modulator. φ can be caused by an I/Q modulator with quadrature error (phase between I and Q channel is not 90 degrees), or by a delay between I and Q of the input signal. In the second case, the resulting *varphi* depends on the frequency of the input signal, according to

$$Q(t) = (1 + \epsilon) \sin\left(\omega_M (t + \Delta t)\right) \quad (13)$$
$$= (1 + \epsilon) \sin\left(\omega_M t + \varphi\right) \quad (14)$$

with

$$\varphi = \omega_M \Delta t \quad (15)$$

Expanding $Q(t)$ and using (12) gives:

$$Q(t) = \sin \omega_M t + \varphi \cos \omega_M t + \epsilon \sin \omega_M t + \varphi\epsilon \;\cos \omega_M t \quad (16)$$

The last term in (16) is of second order nature and can be neglected, so:

$$Q(t) = \sin \omega_M t + \varphi \cos \omega_M t + \epsilon \sin \omega_M t \tag{17}$$

The RF signal is again calculated with:

$$s(t) = \Re \left\{ \left(I(t) + \imath Q(t) \right) e^{\imath \omega_0 t} \right\} \tag{18}$$

This leads to the following result :

$$\begin{aligned} s(t) &= \cos\left(\omega_0 + \omega_M\right) t \\ &\quad - \left(\varphi \cos \omega_M t + \epsilon \sin \omega_M t\right) \sin \omega_0 t \end{aligned} \tag{19}$$

which can be written as:

$$\begin{aligned} s(t) &= \cos\left(\omega_0 + \omega_M\right) t \\ &\quad - \frac{\varphi}{2} \left[\sin\left(\omega_0 + \omega_M\right) t + \sin\left(\omega_0 - \omega_M\right) t\right] \\ &\quad + \frac{\epsilon}{2} \left[\cos\left(\omega_0 + \omega_M\right) t - \cos\left(\omega_0 - \omega_M\right) t\right] \end{aligned} \tag{20}$$

The first terms in the second and third row can be neglected compared to the undisturbed signal (first row), especially if the signal is measured with a spectrum analyzer that usually has a logarithmic scale. Thus:

$$\begin{aligned} s(t) &= \cos\left(\omega_0 + \omega_M\right) t - \frac{\varphi}{2} \sin\left(\omega_0 - \omega_M\right) t - \frac{\epsilon}{2} \cos\left(\omega_0 - \omega_M\right) t \\ &= \cos\left(\omega_0 + \omega_M\right) t - A \sin\left[\left(\omega_0 - \omega_M\right) t + \phi\right] \end{aligned} \tag{21}$$

with:

$$A = \frac{1}{2}\sqrt{\epsilon^2 + \varphi^2} \tag{22}$$

$$\tan \phi = \frac{\varphi}{\epsilon} \tag{23}$$

Note that the disturbances increase with increasing frequency of the input signal if a delay between I and Q of the input signal is present. With (15) follows

$$A = \frac{1}{2}\sqrt{\epsilon^2 + \omega_M^2 \cdot \Delta t^2} \tag{24}$$

$$\tan \phi = \frac{\omega_M \Delta t}{\epsilon} \tag{25}$$